MENIIT
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JEE MAINS-2013

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-CHEMISTRY

1. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism:

(A*) tertiary alcohol by S_N1 (B) secondary alcohol by S_N2

(C) tertiary alcohol by S_N2 (D) secondary alcohol by S_N1

Sol. Tertiary alcohol reacts at fastest rate by S_N1 mechanism

2. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be:

(A*) –5.1 eV (B) –10.2 eV (C) +2.55 eV (D) –2.55 eV

Sol. $Na_{(g)} + 5.1\ eV \longrightarrow Na^+_{(g)} + e^-$

$\therefore$ Ionisation energy of Na = – ΔHeg of Na^+

OR

$Na \longrightarrow Na^+ + e$ $\Delta H = 5.1$ eV

$Na^+ + e \longrightarrow Na$ $\Delta H = -5.1$ eV

3. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of:

(A*) $Li_2^- < Li_2^+ < Li_2$ (B) $Li_2 < Li_2^- < Li_2^+$ (C) $Li_2^- < Li_2 < Li_2^+$ (D) $Li_2 < Li_2^+ < Li_2^-$

Sol. Li_2 has configuration $\sigma(1s)^2 < \sigma^*(1s)^2 < \sigma(2s)^2$

$\therefore$ Bond order of $Li_2 = 1$

Bond order of $Li_2^+ = \frac{1}{2}$

Bond order of $Li_2^- = \frac{1}{2}$ (has greater number of antibonding electrons as compare to Li_2^+ stability α bond order)

4. The molarity of a solution obtained by mixing 750 mL of 0.5 (M) HCl with 250 mL of 2 (M) HCl will be:

(A) 1.00 M (B) 1.75 M (C) 0.975 M (D*) 0.875 M

Sol. 0.5 M HCl HCl 2 M

750 ml 250 ml

$$M = \frac{0.5 \times 750 + 2 \times 250}{1000} = 0.875\ M$$

5. Which of the following is the wrong statement?

(A) O_3 molecule is bent

(B) Ozone is violet-black in solid state

(C) Ozone is diamagnetic gas

(D*) ONCl and ONO^- are not isoelectronic.

Sol. Species having same electronic structure are called isoelectronic species.

6. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E^0_{M^{3+}/M^{2+}}$ value?

(A) Mn (Z = 25) (B) Fe (Z = 26) (C*) Co (Z = 27) (D) Cr (Z = 24)

Sol. $E^0_{M^{3+}/M^{2+}}$

Cr = – 0.41

Mn = 1.57

Fe = 0.77

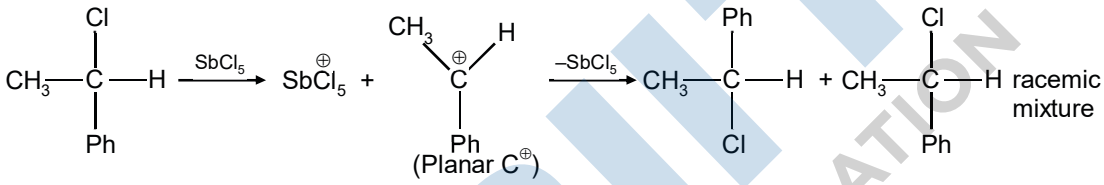
Co = 1.97

[Reference d-block, NCERT page No. 214]

7. A solution of (–) – 1–chloro –1– phenylethane on toluene racemises slowly in the presence of a small amount of $SbCl_5$ due to the formation of :

(A) carbene (B*) carbocation (C) free radical (D) carbanion

Sol.



8. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order :

(A*) $Na^+ < Ba^{2+} < Al^{3+}$ (B) $Ba^{2+} < Na^+ < Al^{3+}$ (C) $Al^{3+} < Na^+ < Ba^{2+}$ (D) $Al^{3+} < Ba^{2+} < Na^+$

Sol. Arsenic Sulphide is negatively charged sol

$Na^+ < Ba^{2+} < Al^{3+}$

9. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?

(A) 0.9 L (B) 2.0 L (C*) 9.0 L (D) 0.1 L

Sol. $10^{-1} \times 1 = 10^{-2} \times V_f$

$\therefore V_f = 10$ L

$\therefore$ Volume of water $(V_f - V_i) = 9$ L

10. Which one of the following molecules is expected to exhibit diamagnetic behaviour?

(A*) N_2 (B) O_2 (C) S_2 (D*) C_2

Sol. $N_2 = \sigma(1s)^2 < \sigma^*(1s)^2 < \sigma(2s)^2 < \sigma^*(2s)^2$

$< \pi(2p_x)^2 = \pi(2p_y)^2 < \sigma(2p_z)^2$

Number of unpaired electron in N_2 and $C_2 = 0$, so both are diamagnetic in nature

Number of unpaired electron in O_2 and S_2 = two, so both are paramagnetic in nature

11. Which of the following arrangements does not represent the correct order of the property stated against it?

(A) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size

(B*) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution

(C) Sc < Ti < Cr < Mn : number of oxidation states

(D*) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour

Sol. On the basis of CFSE, stability of Co^{3+} is highest and stability of Sc^{3+} is minimum among Co^{+3} , Fe^{+3} , Cr^{+3} , Sc^{+3}

Paramagnetic behaviour

$V^{2+} < Cr^{2+} < Fe^{2+} < Mn^{2+}$

12. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be :

(A*) 4.08 % (B) 6.05 % (C) 5.08 % (D) 7.01 %

Sol. $\dfrac{\dfrac{2}{0.98}-2}{1}\times 100$

$\therefore \quad \dfrac{2\times 0.02}{0.98}\times 100 = 4.08\%$ Ans.

13. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is:

(A*) 5 (B) 4 (C) 6 (D) 2

Sol. Molecular mass of compound = 180

Molecular mass after acetylation = 390

Increases in molecular mass = 210

$R\text{–}NH_2 + CH_3\text{–}\overset{\overset{O}{\|}}{C}\text{–}Cl \longrightarrow R\text{–}NH\text{–}\overset{\overset{O}{\|}}{C}\text{–}Me$ increase in molecular mass for one $–NH_2$ group is 42 so total 5 amino groups are present.

14. Given

$E^0_{Cr^{3+}/Cr} = -0.74\,V; E^0_{MnO_4^-/Mn^{2+}} = 1.51V$

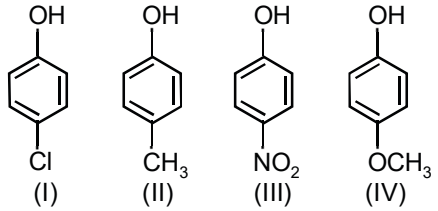
$E^0_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\,V; E^0_{Cl/Cl^-} = 1.36\,V$

Based on the data given above, strongest oxidising agent will be:

(A) Cr^{3+} (B) Mn^{2+} (C*) MnO_4^- (D) Cl^-

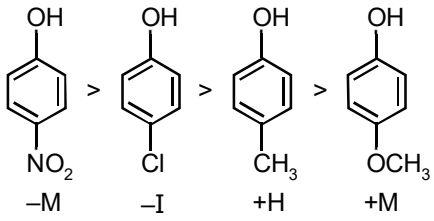
Sol. Higher the value of reduction potential data greater is its tendency to get reduced. Hence substance is a better oxidising agent.

15. Arrange the following compounds in order of decreasing acidity :



(A) I > II > III > IV (B*) III > I > II > IV (C) IV > III > I > II (D) II > IV > I > III

Sol. Acidic strength



Ans. III > I > II > IV

16. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of such a reaction will be: (R = 8.314 JK^{-1} mol^{-1} and log 2 = 0.301)

(A) 48.6 kJ mol^{-1} (B) 58.5 kJ mol^{-1} (C) 60.5 kJ mol^{-1} (D*) 53.6 kJ mol^{-1}

Sol. $$\log\frac{K_2}{K_1}=\frac{E_a}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

$$0.301=\frac{E_a}{2.303\times 8.314}\left[\frac{1}{300}-\frac{1}{310}\right]$$

= 53.6 kJ/mol

17. Synthesis of each molecule of glucose in photosynthesis involves:

(A) 10 molecules of ATP (B) 8 molecules of ATP

(C) 6 molecules of ATP (D*) 18 molecules of ATP

Sol. Number of ATP molecule produce in photosynthesis in light reaction

$12\ H_2O + 12\ NADP + 18\ ADP \rightarrow 16\ CO_2 + 12\ NADPH + 18\ ATP$

Ans. 18 ATP produce to make one glucose

Here

NADP = Nicotimine Adinosine Diphosphate

ADP = Adinosine Diphosphate

NADPH = Nicotinamide Adenine Dinucleotide Phosphate

18. Which of the following complex species is not expected to exhibit optical isomerism?

(A) $[Co(en)_2Cl_2]^+$ (B*) $[Co(NH_3)_3Cl_3]$ (C) $[Co(en)(NH_3)_2Cl_2]^+$ (D) $[Co(en)_3]^{3+}$

Sol. $Co(NH_3)_3Cl_3$ have two fac-mer geometrical isomers are optically inactive.

19. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208 J of heat. The values of q and w for the process will be: (R = 8.314 J / mol K) (ln 7.5 = 2.01)

(A) q = –208 J, w = –208 J (B) q = –208 J, w = +208 J

(C) q = +208 J, w = +208 J (D*) q = +208 J, w = –208 J

Sol. q = 208 J

$\Delta U = 0$ at constant temperature

$\Delta U = q + w$

$w = -q = -208$ J

20. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is:

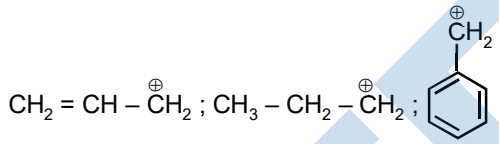
(A) C_3H_4 (B) C_6H_5 (C*) C_7H_8 (D) C_2H_4

Sol. $C_xH_y + O_2 \longrightarrow 7CO_2 + 4H_2O$

C_7H_8 3.08 g 0.72 g

0.07 mole 0.04 mole

21. The order of stability of the following carbocations:

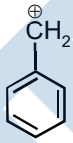


(I) (II) (III)

is :

(A) II > III > I (B) I > II > III (C*) III > I > II (D) III > II > I

Sol. Stability of carbocation



> $CH_2 = CH - \overset{\oplus}{C}H_2$ > $CH_3 - CH_2 - \overset{\oplus}{C}H_2$

better resonance resonance 2 α H

III > I > II

22. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar?

(A) S < Se < Ca < Ba < Ar (B*) Ba < Ca < Se < S < Ar

(C) Ca < Ba < S < Se < Ar (D) Ca < S < Ba < Se < Ar

Sol. Among Ca, Ba, S, Se , Ar

Ba has minimum Ionisation energy and Ar has highest ionisation energy so Ans is (B)

Ba < Ca < Se < S < Ar

23. For gaseous state, if most probable speed is denoted by C*, average speed by $\bar{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speeds are:

(A) C* : $\bar{C}$: C = 1.128 : 1.225 : 1 (B*) C* : $\bar{C}$: C = 1 : 1.128 : 1.225

(C) C* : $\bar{C}$: C = 1 : 1.225 : 1.128 (D) C* : $\bar{C}$: C = 1.225 : 1.128 : 1

Sol. $C^* = \sqrt{\frac{2RT}{M}}; \bar{C} = \sqrt{\frac{8RT}{\pi M}}; C = \sqrt{\frac{3RT}{M}}$

$\sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$

24. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was:

(A) Methylamine (B) Ammonia (C) Phosgene (D*) Methylisocyanate

Sol. (4)
It was methyl isocyanate (CH3NCO)

25. Consider the following reaction:

$$xMnO_4^- + yC_2O_4^{2-} + zH^+ \rightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$

The value of x, y and z in the reaction are, respectively:

(A) 2, 5 and 8 (B*) 2, 5 and 16 (C) 5, 2 and 8 (D) 5, 2 and 16

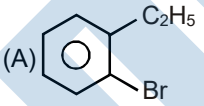
Sol. $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

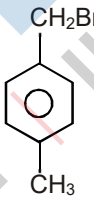
26. Which of the following exists as covalent crystal in the solid state?

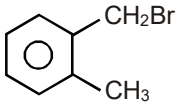
(A*) Silicon (B) Sulphur (C) Phosphorus (D) Iodine

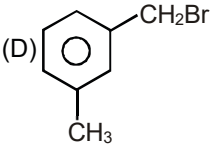
Sol. Si exists as covalent crystal solid, while sulphur (S_8), phosphorus (P_4), Iodine (I_2) exist as molecular crystal solid.

27. Compound (A), C_8H_9Br, gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives an acid (B), $C_8H_6O_4$. (B) easily forms anhydride on heating. Identify the compound (A).

(A) 

(B) 

(C*) 

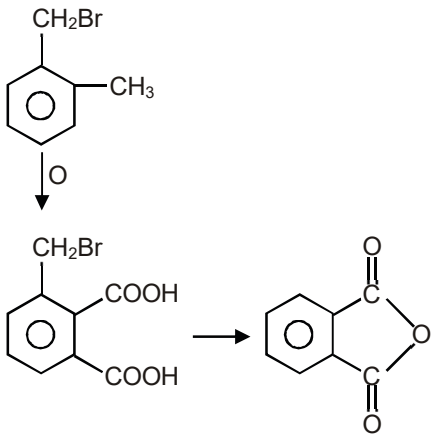
(D) 

Sol. (A) $\xrightarrow{AgNO_3}$ white ppt.

C_8H_9Br

$\downarrow$

(B) $C_8H_6O_4 \xrightarrow{(B)}$ Anhydride



(B) Because it is giving precipitate so Br not attached to ring And anhydrisde form so position is ortho.

28. Energy of an electron is given by $E = -2.178 \times 10^{-18}\ J \left(\frac{Z^2}{n^2}\right)$. Wavelength of light required to excite an electron in an hydrogen atom from level n = 1 to n = 2 will be:

($h = 6.62 \times 10^{-34}$ Js and $c = 3.0 \times 10^{8}\ ms^{-1}$)

(A) 2.816×10^{-7} m (B) 6.500×10^{-7} m (C) 8.500×10^{-7} m (D*) 1.214×10^{-7} m

Sol. $$\frac{hc}{\lambda} = 2.175 \times 10^{-18} Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{2.178 \times 10^{-18} \times 1\left[\frac{1}{1} - \frac{1}{4}\right]}$$

$= 1.214 \times 10^{-7}$ m

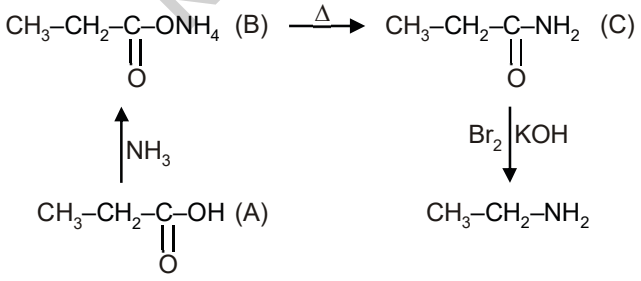
29. An organic compound A upon reacting with NH_3 gives B. On heating B give C, C in presence of KOH reacts with Br_2 to give $CH_3CH_2NH_2$. A is :

(A) $CH_3CH_2CH_2COOH$ (B) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-COOH$

(C*) CH_3CH_2COOH (D) CH_3COOH

Sol. $$A \xrightarrow{NH_3} B \xrightarrow{\Delta} C \xrightarrow[Br_2]{KOH} CH_3CH_2 - NH_2$$

(Hoffmann Bromide)



30. In which of the following pairs of molecules / ions, both the species are not likely to exist?

(A) H_2^-, He_2^{2-} (B*) H_2^{2+}, He_2 (C) H_2^-, He_2^{2+} (D) H_2^+, He_2^{2-}

Sol. H_2^{2+} , He_2 Bond order = 0

so both species are not likely to exist.

PART-B-MATHEMATICS

31. The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point

(A) (2, – 5) (B*) (5, – 2) (C) (– 2, 5) (D) (– 5, 2)

Sol. $\sqrt{(3-1)^2+(y+2)^2} = |y| =$ radius

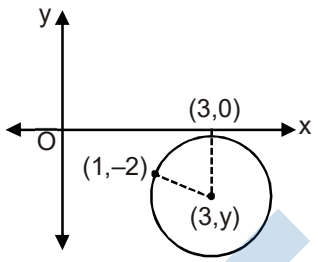
$\Rightarrow$ $4 + (y + 2)^2 = y^2$

$\Rightarrow$ $4y = -8$

$\Rightarrow$ $y = -2$

$\therefore$ Centre $(3, -2)$ and $r = 2$

$\therefore$ $(x - 3)^2 + (y + 2)^2 = 4$



option (B) satisfy.

32. ABCD is a trapezium such that AB and CD are parallel and BC $\perp$ CD. If $\angle ADB = \theta$, BC = p and CD = q, then AB is equal to :

(A) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$ (B) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (C) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$ (D*) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

Sol. From quadrilateral ABCD

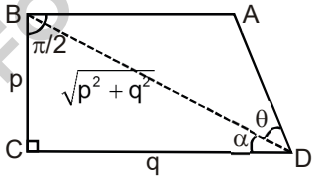
$\frac{\pi}{2}+\frac{\pi}{2}+\theta+\alpha+A = 2\pi$

$\therefore$ $A = (\pi - \theta - \alpha)$

$\therefore$ $\sin A = \sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$

$\sin A = \sin\theta$

$\left(\frac{q}{\sqrt{p^2+q^2}}\right)+\cos\theta\left(\frac{p}{\sqrt{p^2+q^2}}\right)$ (from Δ BCD)



Apply Sine rule in ΔABD

$\frac{AB}{\sin\theta} = \frac{\sqrt{p^2+q^2}}{\sin A}$

$AB = \frac{\sqrt{p^2+q^2}\cdot(\sin\theta)}{\sin A} = \frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

33. Given : A circle $2x^2 + 2y^2 = 5$ and a parabola $y^2 = 4\sqrt{5}x$.

Statement-I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-II: If the line $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(A*) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is false.

(C) Statement-I is false, Statement-II is true.

(D) Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I.

Sol. A tangent of parabola $y^2 = 4\sqrt{5}x$ is

$$y = mx + \frac{\sqrt{5}}{m}(m \neq 0).$$

Since, it touches the circle $x^2 + y^2 = \frac{5}{2}$

$\Rightarrow \frac{\sqrt{5}/m}{\sqrt{1+m^2}} = \sqrt{\frac{5}{2}} \Rightarrow \frac{5}{m^2(1+m^2)} = \frac{5}{2}$

$\Rightarrow m^2(1 + m^2) = 2 \Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow m^2 = 1, -2$

$\Rightarrow m = \pm 1$ which satisfy $m^4 - 3m^2 + 2 = 0$

Hence, common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$. Statement-II is not the correct explanation of Statement-I.

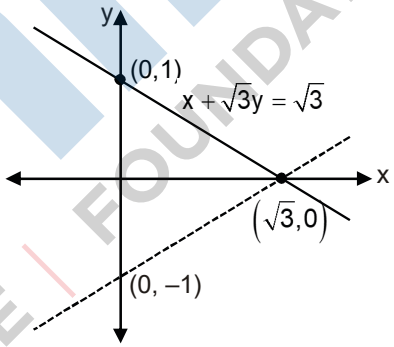
34. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

(A*) $\sqrt{3}y = x - \sqrt{3}$ (B) $y = \sqrt{3}x - \sqrt{3}$ (C) $\sqrt{3}y = x - 1$ (D) $y = x + \sqrt{3}$

Sol. $x + \sqrt{3}y = \sqrt{3}$

Required reflected ray must pass through

$(0, -1)$ and $\left(\sqrt{3}, 0\right)$.



$(y+1) = \frac{0-(-1)}{\sqrt{3}-0}(x-0)$

$\Rightarrow \sqrt{3}y = x - \sqrt{3}$

35. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?

(A) median (B) mode (C*) variance (D) mean

Sol. New mean $\bar{x}' = \bar{x} + 10$ and $x'_i = x_i + 10$

Variance, $(\sigma')^2 = \frac{\sum(\bar{x}' - \bar{x}'_i)^2}{n}$

$= \frac{\sum(\bar{x} - x_i)^2}{n} = \sigma^2$ (unchanged).

36. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then :

(A) $2x = 3y = 6z$ (B) $6x = 3y = 2z$ (C) $6x = 4y = 3z$ (D*) $x = y = z$

Sol. $2y = x + z$ [x, y, z are in A.P.]

$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$ ($\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ in A.P.)

$\Rightarrow \quad \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$ (x + z = 2y)

$\Rightarrow$ $1 - y^2 = 1 - xz$

$\Rightarrow$ $y^2 = xz$

$\Rightarrow$ x, y, z are in G.P. and A.P. both.

Hence x = y = z.

37. If $\int f(x)dx = \Psi(x)$, then $\int x^5 f(x^3)dx$ is equal to :

(A) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^2\Psi(x^3)dx\right] + C$

(B) $\frac{1}{3}x^3\Psi(x^3) - 3\int x^3\Psi(x^3)dx + C$

(C) $\frac{1}{3}x^3\Psi(x^3) - \int x^2\Psi(x^3)dx + C$

(D) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^3\Psi(x^3)dx\right] + C$

Sol. $\int f(x)\,dx = \psi(x)$

$\int x^5 f(x^3)\,dx$

put $x^3 = t \Rightarrow 3x^2dx = dt$

$\frac{1}{3}\int \underbrace{t}_{I}\,\underbrace{f(t)}_{II}\,dt = \frac{1}{3}\left[t\int f(t)\,dt - \int \Psi(t)\,dt\right]$ [Using Integrating by parts]

$= \frac{1}{3}\left[x^3\Psi(x^3) - \int \Psi(x^3)d(x^3)\right]$

$= \frac{1}{3}\left[x^3\Psi(x^3) - \int 3x^2\Psi(x^3)dy\right]$

38. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0, 3) is :

(A) $x^2 + y^2 - 6y + 7 = 0$

(B) $x^2 + y^2 - 6y - 5 = 0$

(C) $x^2 + y^2 - 6y + 5 = 0$

(D*) $x^2 + y^2 - 6y - 7 = 0$

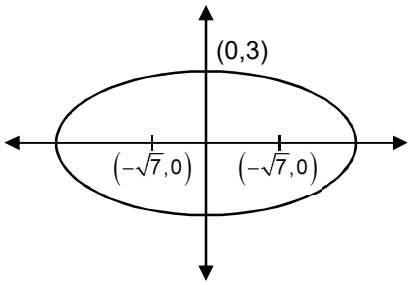
Sol. $r^2 = 7 + 9$

$r^2 = 16$

$r = 4$

$\Rightarrow (x - 0)^2 + (y - 3)^2 = 16$

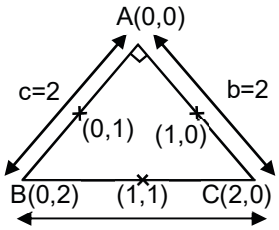
$\Rightarrow x^2 + y^2 - 6y - 7 = 0.$



39. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is :

(A*) $2-\sqrt{2}$ (B) $1+\sqrt{2}$ (C) $1-\sqrt{2}$ (D) $2+\sqrt{2}$

Sol.



From using section formula three vertices are obtained as shown in figure.

Now $x = \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \frac{2\sqrt{2}\times 0 + 2\times 0 + 2\times 2}{2+2+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$ Ans.

40. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\,dt, x \in R$, which are parallel to the line y = 2x are equal to :

(A) ± 2 (B) ± 3 (C) ± 4 (D*) ± 1

Sol. $y = \frac{-x^2}{2} b$ if x < 0

$y = \frac{x^2}{2}$ if $x \geq 0$

$\frac{dy}{dx} = -x$ x < 0

$\frac{dy}{dx} = x$ $x \geq 0$

for slope = 2, x = ± 2

∴ point of tangency (2, 2) (–2, – 2) equation of tangent

(y – (± 2)) = 2(x – (± 2))

for y = 0, x = ± 1 Ans.

41. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is :

(A) $\frac{7}{9}(99 - 10^{-20})$ (B*) $\frac{7}{81}(179 + 10^{-20})$

(C) $\frac{7}{9}(99 + 10^{-20})$ (D) $\frac{7}{81}(179 - 10^{-20})$

Sol. S = 0.7 + 0.77 + 0.777 + ...20 terms

$= \frac{7}{10} + \frac{77}{100} + \frac{777}{1000} + ...20\,\text{terms}$

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{10^2} + \frac{999}{10^3} + ...20\text{ terms}\right]$

$S = \frac{7}{9}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{100}\right) + \left(1-\frac{1}{10^3}\right) + ...20\,\text{terms}\right]$

$= \frac{7}{9}\left[20 - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + ...\frac{1}{10^{20}}\right)\right]$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{1-\frac{1}{10}}\right]$$

$$= \frac{7}{81}\left[179 + 10^{-20}\right]$$ Ans.

42. Consider

Statement-1: $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-2: $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

(A*) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is false.

(C) Statement-I is false, Statement-II is true.

(D) Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I.

Sol. Statement-I: $(p \wedge \sim q) \wedge (\sim p \wedge q) \sim (p \to q) \wedge \sim (q \to p)$

p	q	$\sim (p \to q)$ S	$\sim (q \to p)$ T	$S \wedge T$
T	F	T	F	F
T	T	F	F	F
F	F	F	F	F
F	T	F	T	F

Statement-I is true.

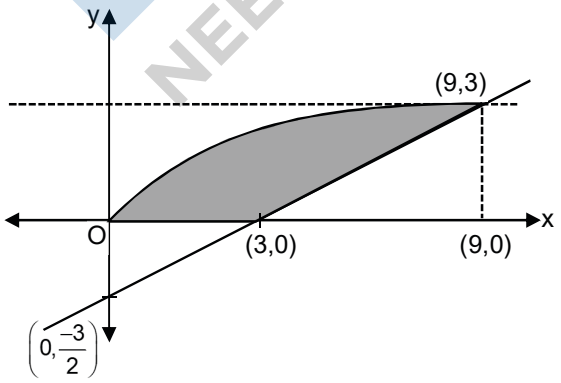
Statement-II: $(p \to q) \leftrightarrow (\sim q \to \sim p)$
$(\sim q \to \sim p)$ is equivalent to $(p \to q)$
$(p \to q) \leftrightarrow (p \to q)$
Hence, $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is tautology.

Statement-II is true but not correct explanation Statement-I.

43. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis and lying in the first quadrant is :

(A) 36 (B) 18 (C) $\frac{27}{4}$ (D*) 9

Sol.



Required area $= \int_0^9 \sqrt{x}\,dx - \frac{1}{2}\times 6\times 3 = 9$ square units.

44. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as :

(A*) sec A cosec A + 1 (B) tan A + cot A (C) sec A + cosec A (D) sin A cos A + 1

Sol. $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=\frac{\tan A}{1-\frac{1}{\tan A}}+\frac{\cot A}{1-\tan A}=\frac{\tan^2 A}{\tan A-1}+\frac{\cot A}{1-\tan A}$

$=\frac{\tan^2 A-\cot A}{\tan A-1}=\frac{\tan^3 A-1}{\tan A\,(\tan A-1)}=\frac{(\tan A-1)(\tan^2 A+\tan A+1)}{\tan A(\tan A-1)}$

$=\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}+1=\frac{\sin^2 A+\cos^2 A}{\cos A\sin A}+1=\sec A\,\text{cosec}\,A+1$

45. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] :

(A) lies between 2 and 3. (B) lies between – 1 and 0.

(C*) does not exist. (D) lies between 1 and 2.

Sol. $f(x) = 2x^3 + 3x + k$

$f'(x) = 6x^2 + 3$

f ' (x) is always positive, hence f (x) is an increasing function

i.e. f (x) has only one real root.

46. $\lim\limits_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to :

(A) $\frac{1}{2}$ (B) 1 (C*) 2 (D) $\frac{-1}{4}$

Sol. $\lim\limits_{x\to 0}\frac{(1-\cos 2x)}{(2x)^2}(2x)^2\cdot(3+\cos x)\cdot\frac{1}{x\left(\frac{\tan 4x}{4x}\right)\cdot 4x}=\frac{1}{2}\cdot(3+1)\cdot\frac{1}{1}=2$ Ans.

47. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is :

(A*) 5 (B) 10 (C) 8 (D) 7

Sol. $T_n = {}^nC_3 \Rightarrow T_{n+1} = {}^{n+1}C_3$

$T_{n+1} - T_n = 10 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 10$

${}^nC_2 = 10$

$\Rightarrow$ n = 5

48. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is :

(A) 3000 (B*) 3500 (C) 4500 (D) 2500

Sol. $\frac{dP}{dx} = 10 - 12\sqrt{x}$

$$\int_{2000}^{P} dP = \int_{0}^{25} (100 - 12\sqrt{x})\, dx$$

$$P - 2000 = 100 \times x - \left.\frac{12x^{\frac{3}{2}}}{3/2}\right|_0^{25}$$

$$= 100x - 8x^{\frac{3}{2}}\Big|_0^{25} = 2500 - 8 \times 125$$

P = 3500. Ans.

49. **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

(A) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is false.

(C*) Statement-I is false, Statement-II is true.

(D) Statement-I is true, Statement-II is true, Statement-II is a correct explanation for Statement-I.

Sol. Let $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx$

By using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx \ ;$$

On adding above two equations, we get

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx = [\, x \,]_{\pi/6}^{\pi/3}$$

$$2I = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}.$$

$$I = \frac{\pi}{12}.$$

Statement-I is false, Statement-II is correct.

50. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and | A | = 4, then α is equal to :

(A*) 11 (B) 5 (C) 0 (D) 4

Sol. | adj A | = | A |^2 = 4^2 = 16

$$\begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix} = 16$$

$\Rightarrow 2\alpha - 6 = 16$

$\Rightarrow \alpha = 11.$

51. The number of values of k, for which the system of equations:

$(k + 1)x + 8y = 4k$

$kx + (k + 3)y = 3k - 1$

has no solution, is :

(A*) 1 (B) 2 (C) 3 (D) infinite

Sol. For no solution

$$\frac{(k+1)}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$$

$\Rightarrow$ From 1^{st} and 2^{nd} ratio

$(k + 1)(k + 3) = 8k$

$\Rightarrow k^2 + 4k + 3 = 8k$

$\Rightarrow k = 1, 3$

For k = 1,

$$\frac{8}{k+3} = 2, \frac{4k}{3k-1} = 2$$

$\Rightarrow$ k = 1 not possible

For k = 3,

$$\frac{8}{k+3} = \frac{4}{3}, \frac{4k}{3k-1} = \frac{3}{2}$$

$\Rightarrow$ k = 3 is possible.

52. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at x = 1 is equal to :

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D*) $\frac{1}{\sqrt{2}}$

Sol. $y = \sec(\tan^{-1}x)$

$$y = \sec\left(\sec^{-1}\sqrt{1+x^2}\right)$$

$$y = \sqrt{1+x^2}$$

$$\left(\frac{dy}{dx}\right)_{x=1} = \left(\frac{2x}{2\sqrt{1+x^2}}\right)_{x=1} = \frac{1}{\sqrt{2}}.$$ Ans.

53. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have :

(A) exactly one value (B*) exactly two values

(C) exactly three values (D) any value

Sol. If the lines are coplanar

$$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \quad \text{or} \quad k^2 + 3k = 0$$

$\Rightarrow$ k = 0, – 3.

54. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of A × B having 3 or more elements is :

(A) 220 (B*) 219 (C) 211 (D) 256

Sol. $A \rightarrow 2$ elements

$B \rightarrow 4$ elements

$n(A \times B) = 2 \times 4 = 8$

$\therefore$ number of subset having 3 or more element

$= {}^8C_8 + {}^8C_7 + {}^8C_6 + {}^8C_5 + {}^8C_4 + {}^8C_3$

$= 2^8 - ({}^8C_2 + {}^8C_1 + {}^8C_0)$

$= 256 - [28 + 8 + 1]$

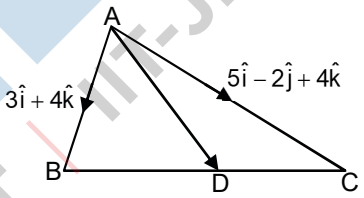
$= 219$ Ans.

55. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is :

(A) $\sqrt{72}$ (B*) $\sqrt{33}$ (C) $\sqrt{45}$ (D) $\sqrt{18}$

Sol. $\left|\overrightarrow{AD}\right| = \left|\frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}\right|$

$= \left|\frac{8\hat{i} - 2\hat{j} + 8\hat{k}}{2}\right|$



$= \sqrt{33}$ Ans.

56. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is :

(A) $\frac{13}{3^5}$ (B*) $\frac{11}{3^5}$ (C) $\frac{10}{3^5}$ (D) $\frac{17}{3^5}$

Sol. Favourable case = 4 correct + 5 correct

$$= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^5$$

$$= 10\left(\frac{1}{3}\right)^5 + \left(\frac{1}{3}\right)^5$$

$$= \frac{11}{3^5} \quad \text{Ans.}$$

Aliter: $n(S) = 3^5$; $n(A) = {}^5C_4 \times 2 + {}^5C_5$

Required probability $= \dfrac{{}^5C_4 \times 2 + 1}{3^5}$

57. If z is a complex number of unit modulus and argument θ, then arg $\left(\dfrac{1+z}{1+\bar{z}}\right)$ equals :

(A) $\dfrac{\pi}{2} - \theta$ (B*) θ (C) $\pi - \theta$ (D) $-\theta$

Sol. $|z| = 1 \Rightarrow |z|^2 = 1 \Rightarrow z\bar{z} = 1$

$$\arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{1+z}{1+\frac{1}{z}}\right) = \arg(z) = \theta$$

58. If the equation $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a, b, c $\in$ R, have a common root, then a : b : c is :

(A) 3 : 2 : 1 (B) 1 : 3 : 2 (C) 3 : 1 : 2 (D*) 1 : 2 : 3

Sol. For $x^2 + 2x + 3 = 0$, D < 0

$\Rightarrow$ both equations has both roots common.

$$\Rightarrow \frac{a}{1} = \frac{b}{2} = \frac{c}{3}$$

59. Distance between two parallel planes, 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is :

(A) $\dfrac{5}{2}$ (B*) $\dfrac{7}{2}$ (C) $\dfrac{9}{2}$ (D) $\dfrac{3}{2}$

Sol. $P_1 = 2x + y + 2z - 8 = 0$

$P_2 = 2x + y + 2z - \dfrac{5}{2} = 0$

$$d = \left|\frac{8 + \frac{5}{2}}{\sqrt{4+1+4}}\right| = \frac{21}{6} = \frac{7}{2} \quad \text{Ans.}$$

60. The term independent of x in expansion of $\left(\dfrac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \dfrac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$ is :

(A) 120 (B*) 210 (C) 310 (D) 4

Sol. $\left(\frac{\left(x^{1/3}+1\right)\left(x^{2/3}-x^{1/3}+1\right)}{x^{2/3}-x^{1/3}+1}-\frac{\left(x^{1/2}-1\right)\left(x^{1/2}+1\right)}{x^{1/2}\left(x^{1/2}-1\right)}\right)^{10}$

$=\left(\left(x^{1/3}+1\right)-\left(1+x^{-1/2}\right)\right)^{10}$

$=\left(x^{1/3}-\frac{1}{x^{1/2}}\right)^{10}$

Let r + 1 is independent term

$\therefore \quad T_{r+1} = {}^{10}C_r\,(x^{1/3})^{10-r}\left(\frac{-1}{x^{1/2}}\right)^r = (-1)^r \cdot {}^{10}C_r\; x^{\frac{10-r}{3}-\frac{r}{2}}$

$\therefore$ for independent term $\frac{10-r}{3}-\frac{r}{2}=0$

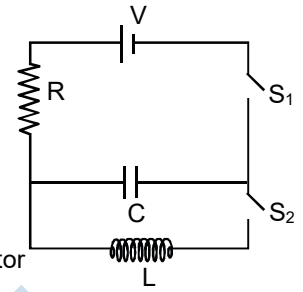
$\Rightarrow \quad r = 4$

$\therefore \quad 5^{th}$ term is independent

$\therefore \quad T_5 = T_{4+1} = (-1)^4 \cdot {}^{10}C_4 = 210$ Ans.

PART-C-PHYSICS

61. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct ?



(A) At $t = \tau$, $q = CV/2$

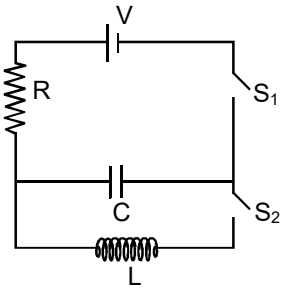
(B*) At $t = 2\tau$, $q = CV(1- e^{-2})$

(C) At $t = \frac{\tau}{2}$, $q = CV(1- e^{-1})$

(D) Work done by the battery is half of the energy dissipated in the resistor

Sol. $q = q_0 (1 - e^{-t/\tau})$

$= CV(1 - e^{-2\tau/\tau})$



62. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

(A*) 10.62 kHz (B) 5.31 MHz (C) 5.31 kHz (D) 10.62 MHz

Sol. $\frac{1}{f_c} << Rc << \frac{1}{f_M}$

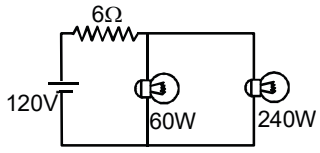
$\Rightarrow f_M << \frac{1}{Rc} = 40\,kHz$

of 4 option 10.62 kHz is max.

63. The supply voltage to a room is 120 V. The resistance of the lead wires is 6Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb ?

(A) 2.9 Volt (B) 13.3 Volt (C*) 10.04 Volt (D) zero Volt

Sol.



$R_B = \frac{V^2}{P} = \frac{120 \times 120}{60} = 240\ \Omega$

$R_H = \frac{V^2}{P} = \frac{120 \times 120}{240} = 60\Omega$

Initially $V_B = \frac{120}{246} \times 240 = 117$ V

Later on $V_B = \frac{120}{54} \times 48 = 106.6$ V

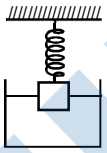
64. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is :

(A) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (B*) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (C) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$ (D) $\frac{Mg}{k}$

(Here k is spring constant)

Sol. $Mg = kx_0 + \frac{\sigma AL}{2}g$

$\Rightarrow x_0 = \frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$



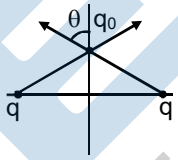
65. Two charges, each equal to q, are kept at x = –a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y << a) along the y-axis, the net force acting on the particle is proportional to :

(A) –y (B) $\frac{1}{y}$ (C) $-\frac{1}{y}$ (D*) y

Sol. $F = 2F_1 \cos\theta$

$= \frac{2 \times kq_0 q}{(y^2 + a^2)^{3/2}} y$

$\Rightarrow F \propto y$



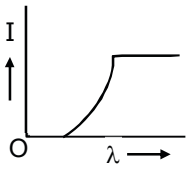
66. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is:

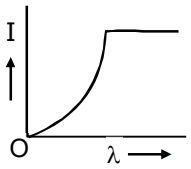
(A) $I_0/2$ (B*) $I_0/4$ (C) $I_0/8$ (D) I_0

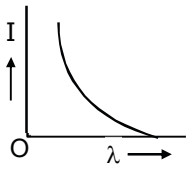
Sol. $I_1 = \frac{I_0}{2}$

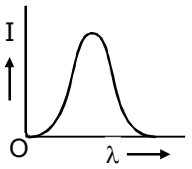
$I_2 - I_1 \cos^2\theta = \frac{I_0}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2$

67. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows :

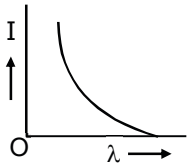
(A) 

(B) 

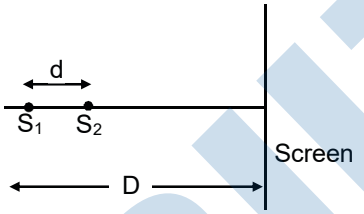
(C*) 

(D) 

Sol. Beyond threshold wavelength current become zero.



68. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be :

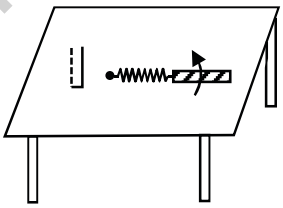


(A) straight lines (B) semi-circles (C*) concentric circles (D) points

Sol.

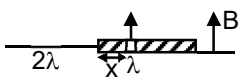
Locus of the points where the path difference is same will be a circle in this case.

69. A metallic rod of length 'l ' is tied to a string of length 21 and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is:



(A) $\frac{3B\omega\ell^2}{2}$ (B) $\frac{4B\omega\ell^2}{2}$ (C*) $\frac{5B\omega\ell^2}{2}$ (D) $\frac{2B\omega\ell^2}{2}$

Sol. $d\varepsilon = (\vec{V}\times\vec{B})d\vec{\ell} = \int_{2\ell}^{3\ell} \omega x B dx$



$= \omega B\left(\frac{x^2}{2}\right)_{2\ell}^{3\ell}$

$= \frac{\omega B}{2}(9\ell^2 - 4\ell^2) = \frac{5\omega B\ell^2}{2}$

70. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n – 1). If n >>1, the frequency of radiation emitted is proportional to :

(A) $\frac{1}{n^2}$ (B) $\frac{1}{n^{3/2}}$ (C*) $\frac{1}{n^3}$ (D) $\frac{1}{n}$

Sol. $\frac{1}{\lambda} = R\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$

$\nu = RC\left(\frac{n^2-(n-1)^2}{(n-1)^2n^2}\right) = \frac{2n-1}{(n-1)^2n^2} = \frac{2n}{n^4} \propto \frac{1}{n^3}$

71. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(A) $\sqrt{T/\rho L}$ (B) $T/\rho L$ (C*) $2T/\rho L$ (D) $\rho L/T$

Sol. $\frac{\Delta Q}{\Delta t} = \frac{dm}{dt}L$ & $\frac{dE}{dt} = 4\pi T2R\frac{dr}{dt}$

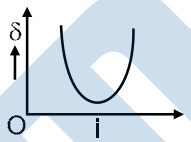
$\frac{dm}{dt}L = 4\pi T2R\frac{dr}{dt}$

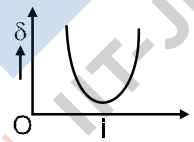
$\frac{d}{dt}(\rho\frac{4}{3}\pi r^3)L = 8\pi TR\frac{dr}{dt}$

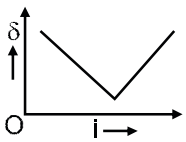
$\rho\frac{4}{3}3r^2\frac{dr}{dt}L = 8pT\frac{dr}{dt}$

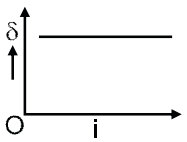
$r = \frac{2T}{\rho L}$

72. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by :

(A) 

(B*) 

(C) 

(D) 

73. Let $[\in_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then :

(A*) $[\in_0] = [M^{-1}L^{-3}T^4A^2]$ (B) $[\in_0] = [M^{-1}L^2T^{-1}A^{-2}]$

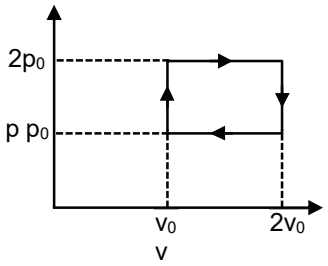
(C) $[\in_0] = [M^{-1}L^2T^{-1}A]$ (D) $[\in_0] = [M^{-1}L^{-3}T^2A]$

Sol. $M^1L^1T^{-2}\frac{I^2T^2}{[\varepsilon_0]L^2}$

$[\varepsilon_0] = \frac{I^2T^2L^{-2}}{M^1L^1T^{-2}}$

$[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$

74.



The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle, is :

(A*) $\left(\frac{13}{2}\right)p_0v_0$ (B) $\left(\frac{11}{2}\right)p_0v_0$ (C) $4p_0v_0$ (D) p_0v_0

Sol. $Q_{\text{given during isochoric \& isobaric process}}$

$$Q_1 = nCv\,\Delta T_1 = \frac{3}{2}nR\Delta T_1 = \frac{3}{2}P_0V_0$$

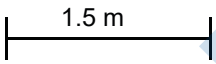
$$Q_2 = nC_P\,\Delta T_2 = \frac{5}{2}\,nR\Delta T_2 = \frac{5}{2}\times 2P_0\times V_0$$

$$Q = Q_1 + Q_2 = \frac{13}{2}P_0V_0$$

75. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?

(A*) 178.2 Hz (B) 200.5 Hz (C) 770 Hz (D) 188.5 Hz

Sol.



$$f = \frac{V}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\frac{T}{\mu} = \frac{y}{\rho}\times\frac{\Delta\ell}{\ell}$$

$$\Rightarrow f = \frac{1}{2\times1.5}\sqrt{\frac{2.2\times10^{11}\times0.01}{7.7\times10^3}}$$

$$= \frac{1}{3}\sqrt{\frac{2}{7}\times10^6}$$

$$= \frac{1}{3}\sqrt{\frac{2}{7}}\times103 = 178.2\text{ Hz}$$

76. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-I: Higher the range, greater is the resistance of ammeter.

Statement-II: To increase the range of ammeter, additional shunt needs to be used across it.

(A) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

(B) Statement-I is true, Statement-II is false.

(C*) Statement-I is false, Statement-II is true.

(D) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.

Sol. $i_g \times G = I \times \frac{RG}{R+G}$

$\Rightarrow I = i_g\left(1+\frac{G}{R}\right)$

77. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

(A) $\frac{2GmM}{3R}$ (B) $\frac{GmM}{2R}$ (C) $\frac{GmM}{3R}$ (D*) $\frac{5GmM}{6R}$

Sol. Required energy $-\frac{GMm}{R} = \frac{-GMm}{6R}$

78. A projectile is given an initial velocity of $\left(\hat{i}+2\hat{j}\right)$m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is :

(A*) $y = 2x - 5x^2$ (B) $4y = 2x - 5x^2$ (C) $4y = 2x - 25x^2$ (D) $y = x - 5x^2$

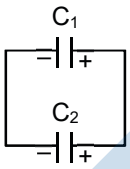
Sol. $x = t$

$y = 2t - 5t^2$

79. Two capacitors C_1 and C_2 are charged to 120V and 200V respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(A*) $3C_1 = 5C_2$ (B) $3C_1 + 5C_2 = 0$ (C) $9C_1 = 4C_2$ (D) $5C_1 = 3C_2$

Sol.



$C_1 \times 120 = C_2 \times 200$

80. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(A) $\frac{r\omega_0}{3}$ (B*) $\frac{r\omega_0}{2}$ (C) $r\omega_0$ (D) $\frac{r\omega_0}{4}$

Sol. $mr^2\omega_0 = mvr + mr^2\frac{v}{r}$

$m\omega_0 = \frac{2Mv}{r}$

$v = \frac{\omega_0 r}{2}$

81. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency.

(A) $\frac{1}{2\pi}\frac{V_0MP_0}{A^2\gamma}$ (B*) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (D) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0M}$

Sol. $\frac{\Delta P}{\Delta V}=\frac{\gamma P}{V}$

$\Delta P=\frac{\gamma P}{V}\Delta V$

$F=\Delta PA=\frac{\gamma P\Delta VA}{V}=\frac{\gamma PA^2\Delta x}{V}$

$a=\frac{f}{m}=\frac{\gamma PA^2x}{Vm}=\omega^2x$

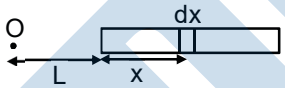
$\omega=\sqrt{\frac{\gamma PA^2}{VM}}$

$f=\frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV}}$

82. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is :

(A) $\frac{3Q}{4\pi\varepsilon_0L}v$ (B) $\frac{Q}{4\pi\varepsilon_0L\ln 2}$ (C*) $\frac{Q\ell n2}{4\pi\varepsilon_0L}$ (D) $\frac{Q}{8\pi\varepsilon_0L}$

Sol.



$V=\int_0^L k\frac{Q}{L}\frac{dx}{(L+x)}$

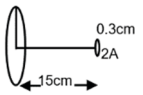
$=\frac{kQ}{L}\left[\ell n(L+x)\right]_0^L$

$=\frac{kQ}{L}\ell n\frac{2L}{2L}=\frac{kQ\ell n^2}{L}$

83. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is :

(A) 6×10^{-11} weber (B) 3.3×10^{-11} weber (C) 6.6×10^{-9} weber (D*) 9.1×10^{-11} weber

Sol. $B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}$



$B = \frac{4\pi \times 10^{-7} \times i(0.2)^2}{2(1.5625 \times 10^{-2})}$

$\phi = BA = \frac{4\pi \times 10^{-7} \times 0.04}{2(1.5625 \times 10^{-2})} \times \pi(0.03)^2$

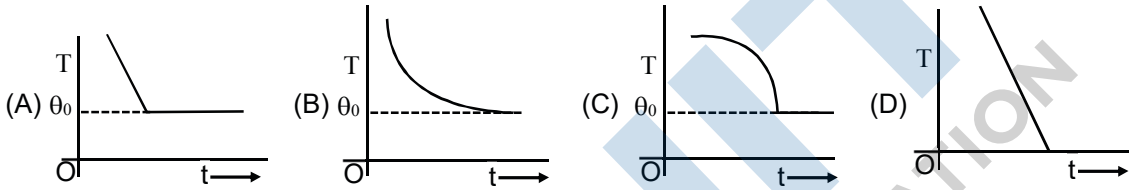
$= 4.6 \times 10^{-6} \times 10^{-7} \times 10^2 = M^{12}$

$= 4.6 \times 10^{-11}$

$M_{12} = M_{21}$

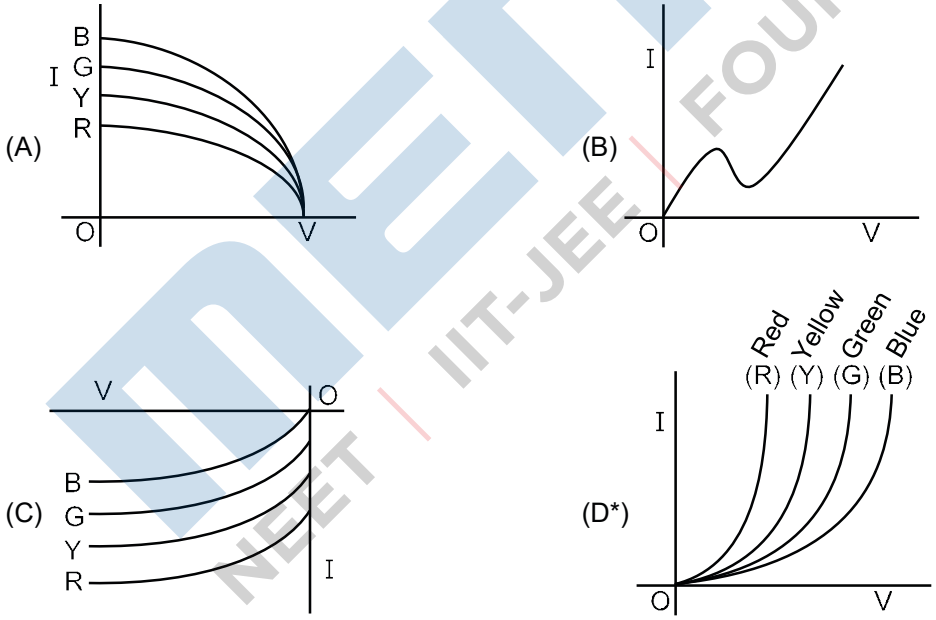
$\phi = 4.6 \times 10^{-11} \times 2 = 9.2 \times 10^{-11}$ weber

84. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to :



Sol. Newton's Law of cooling.

85. The I-V characteristic of an LED is :



Sol. (4)

For LED, in forward bias, intensity increases with voltage.

86. This question has statement-I and statement-II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-I: A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as then $f\left(\frac{1}{2}mv^2\right)$.

Statement-II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(A) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation of Statement-I

(B) Statement-I is true, Statement-II is false

(C*) Statement-I is false, Statement-II is true

(D) Statement-I is true, Statement-II is true, Statement-II is a correct explanation of Statement-I.

Sol. $\Delta E = \frac{1}{2}\mu(v_{rel}^2 - u_{rel}^2) = \frac{1}{2}\frac{mM}{m+M}(u_{rel}^2)$

87. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10 s it will decrease to α times its original magnitude, where α equals :

(A) 0.81 (B*) 0.729 (C) 0.6 (D) 0.7

Sol. $A = A_0e^{-rt}$

$0.9A_0 = A_0e^{-r\times 5}$

$\alpha A_0 = A_0e^{-r\times 15}$

$\Rightarrow \alpha = (0.9)^3 = 0.729$

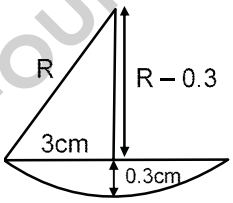
88. Diameter of a plano - convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is :

(A) 20 cm (B*) 30 cm (C) 10 cm (D) 15 cm

Sol. $3^2 + (R - 0.3)^2 = R^2$

$3^2 + R^2 - 0.6R + 0.09 = R^2$

$\Rightarrow R = \frac{3^2}{0.6} = 15$ cm



$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{1}{30}$

89. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

(A*) 6 V/m (B) 9 V/m (C) 12 V/m (D) 3 V/m

Sol. $E_0 = CB_0$

$E_0 = 3 \times 10^8 \times 20 \times 10^{-9}$

$= 60 \times 10^{-1} = 6$V/m

90. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

(A*) 2.56×10^{-4} Wb/m^2 (B) 3.50×10^{-4} Wb/m^2 (C) 5.80×10^{-4} Wb/m^2 (D) 3.6×10^{-5} Wb/m^2

Sol. $B_1 = \frac{\mu_0 M_1 \sin 90°}{4\pi r^3}$

$B_2 = \frac{\mu_0 M_2 \sin 90°}{4\pi r^3}$

$B = B_1 + B_2 + B_{ext}$

$= \frac{\mu_0}{4\pi \times (0.1)^2} (1.2 + 1) + 3.6 \times 10^{-5}$

$= 10^{-4} \times 2.2 + 3.6 \times 10^{-5}$

$= 25.6 \times 10^{-5}$ wb/m^2